Theoretical Study and Numerical Analysis of Near 3D Sound Field Reproduction Based on Wave Field Synthesis

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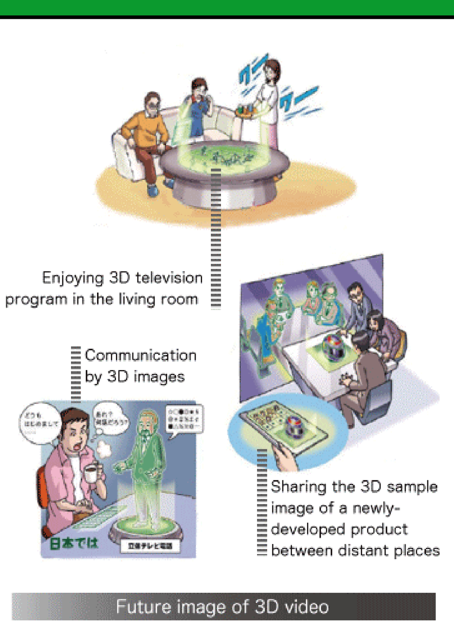
1. INTRODUCTION

Ultra-Realistic Communication

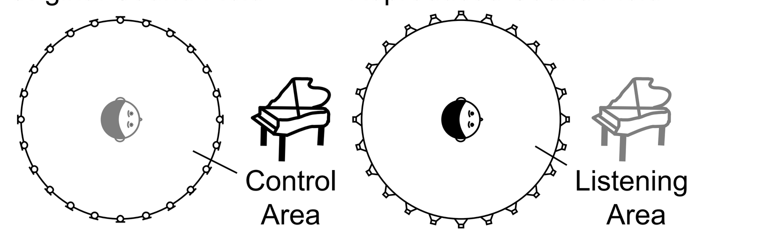
- More realistic form of communication
  - 3D television
  - 3D teleconferencing
- 3D sound field reproduction
  - Binaural
  - Transaural
  - Stereo dipole
  - Wave field synthesis
  - Boundary surface control

We focus on this technique

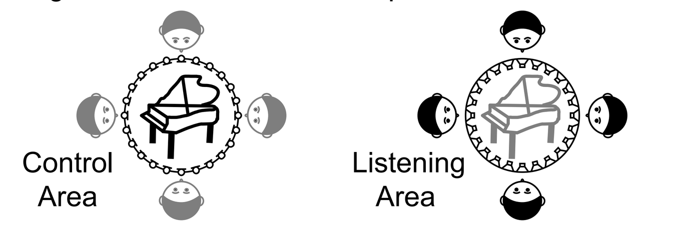


Wave Field Synthesis

- Conventional system
  - Loudspeakers are placed around listeners
  - Listeners cannot listen to sounds around sound sources
  - Theory has been studied



- Proposed system
  - Loudspeakers are placed around sound sources
  - Listeners can listen to sounds around sound sources
  - **Theory has not been studied**



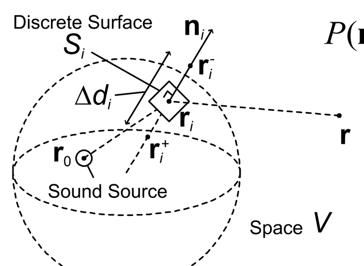
Aim of Study

- Near 3D sound field reproduction technique based on wave field synthesis is proposed...Two methods (dipole control method and directional point control method)

2. PRINCIPLE OF NEAR 3D SOUND FIELD REPRODUCTION

2.1. Wave Field Synthesis

- Kirchhoff-Helmholtz integral equation
  - Sound pressures of space V is reproduced if two types of sources are played at M points $\mathbf{r}_i$
    - Monopole source (amplitude $\partial P(\mathbf{r}_i,\omega)/\partial \mathbf{n}_i$)
    - Dipole source (amplitude $-P(\mathbf{r}_i,\omega)$)



$$P(\mathbf{r},\omega)=\frac{1}{4\pi}\sum_{i=1}^{M}\left\{\frac{\partial P(\mathbf{r}_i,\omega)}{\partial \mathbf{n}_i}\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}-P(\mathbf{r}_i,\omega)\frac{\partial}{\partial \mathbf{n}_i}\left(\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\right)\right\}\Delta S_i$$

k: Wave number
$\mathbf{r}$: Position vector on space V
$\mathbf{r}_i$: Position vector on S_i
$\mathbf{n}_i$: Normal unit vector at $\mathbf{r}_i$

2.2. Dipole Control Method

- Approximation is introduced
  - Sound pressure gradient

$$\frac{\partial P(\mathbf{r}_i,\omega)}{\partial \mathbf{n}_i}\approx\frac{P(\mathbf{r}_i^+,\omega)-P(\mathbf{r}_i^-,\omega)}{\Delta d_i}$$

  - Sound pressure

$$P(\mathbf{r}_i,\omega)\approx\frac{P(\mathbf{r}_i^+,\omega)+P(\mathbf{r}_i^-,\omega)}{2}$$

  - Dipole source

$$\frac{\partial}{\partial \mathbf{n}_i}\left(\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\right)\approx\frac{1}{\Delta d_i}\left(\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i^+|}}{|\mathbf{r}-\mathbf{r}_i^+|}-\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i^-|}}{|\mathbf{r}-\mathbf{r}_i^-|}\right)$$

  - Monopole source

$$\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\approx\frac{1}{2}\left(\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i^+|}}{|\mathbf{r}-\mathbf{r}_i^+|}+\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i^-|}}{|\mathbf{r}-\mathbf{r}_i^-|}\right)$$

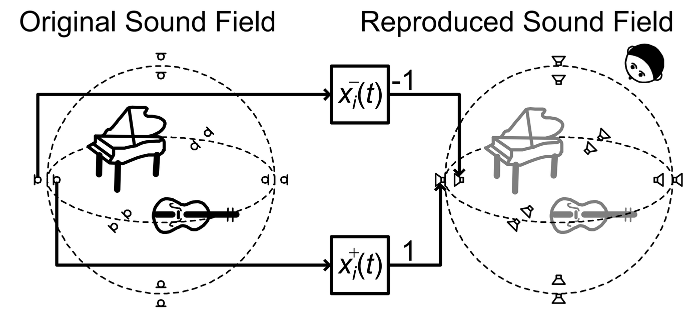
- Kirchhoff-Helmholtz integral equation
  - Sound pressures of space V is reproduced if two types of sources are played at the neighbor of M points $\mathbf{r}_i$
    - Monopole source (position $\mathbf{r}_i^-$, amplitude $P(\mathbf{r}_i^+,\omega)$)
    - Monopole source (position $\mathbf{r}_i^+$, amplitude $-P(\mathbf{r}_i^-,\omega)$)

$$P(\mathbf{r},\omega)=\frac{1}{4\pi}\sum_{i=1}^{M}\left\{P(\mathbf{r}_i^+,\omega)\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i^-|}}{|\mathbf{r}-\mathbf{r}_i^-|}-P(\mathbf{r}_i^-,\omega)\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i^+|}}{|\mathbf{r}-\mathbf{r}_i^+|}\right\}\frac{\Delta S_i}{\Delta d_i}$$

k: Wave number
$\mathbf{r}$: Position vector on space V
$\mathbf{r}_i^-$: Position vector of the outside neighbor point on S_i
$\mathbf{r}_i^+$: Position vector of the inside neighbor point on S_i

Diagram of Dipole Control Method

1. Sounds are recorded by microphone pairs
2. Sounds are played by loudspeaker pairs
3. Wave fronts are reproduced on the outside of loudspeaker array
4. Listeners feel that sound is playing on the inside of loudspeaker array



2.3. Directional Point Control Method

- Approximation is introduced
  - Sound pressure gradient

$$\frac{\partial P(\mathbf{r}_i,\omega)}{\partial \mathbf{n}_i}=\frac{\partial}{\partial \mathbf{n}_i}\left(\frac{Ae^{-jk|\mathbf{r}_i-\mathbf{r}_0|}}{|\mathbf{r}_i-\mathbf{r}_0|}\right)$$

$$=-\frac{Ae^{-jk|\mathbf{r}_i-\mathbf{r}_0|}}{|\mathbf{r}_i-\mathbf{r}_0|}\left(\frac{1}{|\mathbf{r}_i-\mathbf{r}_0|}+jk\right)\cos\theta_{i0}$$

$$=-P(\mathbf{r}_i,\omega)\left(\frac{1}{|\mathbf{r}_i-\mathbf{r}_0|}+jk\right)\cos\theta_{i0}$$

$$\approx -jkP(\mathbf{r}_i,\omega)\cos\theta_{i0}\left(\text{if } k>>\frac{1}{|\mathbf{r}_i-\mathbf{r}_0|}\right)$$

  - Dipole source

$$\frac{\partial}{\partial \mathbf{n}_i}\left(\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\right)=-\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\left(\frac{1}{|\mathbf{r}-\mathbf{r}_i|}+jk\right)\cos\theta_i$$

$$\approx -jk\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\cos\theta_i\left(\text{if } k>>\frac{1}{|\mathbf{r}-\mathbf{r}_i|}\right)$$

$\cos\theta_{i0}$: Angle between vectors $\mathbf{n}_i$ and $\mathbf{r}_i$-$\mathbf{r}_0$
$\cos\theta_i$: Angle between vectors $\mathbf{n}_i$ and $\mathbf{r}_i$-$\mathbf{r}$

$$\cos\theta_{i0}=\frac{\mathbf{n}_i\cdot(\mathbf{r}_i-\mathbf{r}_0)}{|\mathbf{n}_i\|\mathbf{r}_i-\mathbf{r}_0|}\qquad \cos\theta_i=\frac{\mathbf{n}_i\cdot(\mathbf{r}_i-\mathbf{r})}{|\mathbf{n}_i\|\mathbf{r}_i-\mathbf{r}|}$$

- Fresnel-Kirchhoff diffraction formula
  - Sound pressures of space V is reproduced if directional monopole sources are played at M points $\mathbf{r}_i$
    - Monopole source (directivity D_i, amplitude $P(\mathbf{r}_i,\omega)$)

$$P(\mathbf{r},\omega)=\frac{jk}{4\pi}\sum_{i=1}^{M}P(\mathbf{r}_i,\omega)\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}(\cos\theta_i-\cos\theta_{i0})\Delta S_i$$

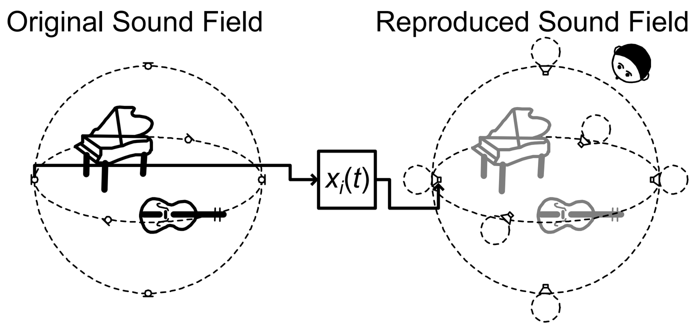
$$\approx\frac{jk}{4\pi}\sum_{i=1}^{M}P(\mathbf{r}_i,\omega)\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}(\cos\theta_i-1)\Delta S_i$$

$$\approx\frac{jk}{4\pi}\sum_{i=1}^{M}D_iP(\mathbf{r}_i,\omega)\frac{e^{-jk|\mathbf{r}-\mathbf{r}_i|}}{|\mathbf{r}-\mathbf{r}_i|}\Delta S_i$$

D_i: Directivity of loudspeakers

Diagram of Directional Point Control Method

1. Sounds are recorded by microphones
2. Sounds are played by directional loudspeakers
3. Wave fronts are reproduced on the outside of loudspeaker array
4. Listeners feel that sound is playing on the inside of loudspeaker array



3. COMPUTER SIMULATION

3.1. Simulation Environment

Original Sound Field

- Source signal $s(t)$
  - Sine-wave (amplitude A, frequency f)

$$s(t)=A\sin 2\pi ft$$

- Sound pressure $p_0(\mathbf{r}, f, t)$

$$p_0(\mathbf{r},f,t)=\frac{1}{d_0}s\left(t-\frac{d_0}{c}\right)=\frac{A}{d_0}\sin\left\{2\pi f\left(t-\frac{d_0}{c}\right)\right\}$$

$\mathbf{r}$: Position vector of synthesis points
d_0(=|$\mathbf{r}$−$\mathbf{r}_0$|): Distance between sound sources and synthesis points
$\mathbf{r}_0$: Position vector of sound sources
c: Sound velocity

Dipole Control Method

- Recorded signals $x_i^+(t)$, $x_i^-(t)$

$$x_i^+(t)=\frac{1}{d_{i0}^+}s\left(t-\frac{d_{i0}^+}{c}\right)=\frac{A}{d_{i0}^+}\sin\left\{2\pi f\left(t-\frac{d_{i0}^+}{c}\right)\right\}$$

$$x_i^-(t)=\frac{1}{d_{i0}^-}s\left(t-\frac{d_{i0}^-}{c}\right)=\frac{A}{d_{i0}^-}\sin\left\{2\pi f\left(t-\frac{d_{i0}^-}{c}\right)\right\}$$

d_{i0}^+(=|$\mathbf{r}_i^+$−$\mathbf{r}_0$|), d_{i0}^-(=|$\mathbf{r}_i^-$−$\mathbf{r}_0$|): Distance between sound sources and microphone pairs
$\mathbf{r}_i^+$, $\mathbf{r}_i^-$: Position vector of microphone pairs

$$\mathbf{r}_i^+=\mathbf{r}_i-\frac{\Delta d_i}{2}\mathbf{n}_i,\quad \mathbf{r}_i^-=\mathbf{r}_i+\frac{\Delta d_i}{2}\mathbf{n}_i$$

- Sound pressure $p(\mathbf{r}, f, t)$

$$p(\mathbf{r},f,t)=\sum_{i=1}^{M}\left\{\frac{1}{d_i^-}x_i^+\left(t-\frac{d_i^-}{c}\right)-\frac{1}{d_i^+}x_i^-\left(t-\frac{d_i^+}{c}\right)\right\}$$

$$=\sum_{i=1}^{M}\left[\frac{A}{d_i^-d_{i0}^+}\sin\left\{2\pi f\left(t-\frac{d_i^-+d_{i0}^+}{c}\right)\right\}-\frac{A}{d_i^+d_{i0}^-}\sin\left\{2\pi f\left(t-\frac{d_i^++d_{i0}^-}{c}\right)\right\}\right]$$

M: Total number of loudspeaker pairs
d_i^+(=|$\mathbf{r}$−$\mathbf{r}_i^+$|), d_i^-(=|$\mathbf{r}$−$\mathbf{r}_i^-$|): Distance between loudspeaker pairs and synthesis points

Directional Point Control Method

- Recorded signals $x_i(t)$

$$x_i(t)=\frac{1}{d_{i0}}s\left(t-\frac{d_{i0}}{c}\right)=\frac{A}{d_{i0}}\sin\left\{2\pi f\left(t-\frac{d_{i0}}{c}\right)\right\}$$

- Sound pressure $p(\mathbf{r}, f, t)$

$$p(\mathbf{r},f,t)=\sum_{i=1}^{M}\frac{D_i}{d_i}x_i\left(t-\frac{d_i}{c}\right)=\sum_{i=1}^{M}\frac{D_iA}{d_id_{i0}}\sin\left\{2\pi f\left(t-\frac{d_i+d_{i0}}{c}\right)\right\}$$

d_{i0}(=|$\mathbf{r}_i$−$\mathbf{r}_0$|): Distance between sound sources and microphones
$\mathbf{r}_i$: Position vector of microphones
M: Total number of loudspeakers
d_i(=|$\mathbf{r}$−$\mathbf{r}_i$|): Distance between loudspeakers and synthesis points
D_i: Directivity of loudspeakers

Parametric Condition

Source amplitude (A)	1
Source frequency (f)	125, 250, 500, 1000, 2000, 4000, 8000, 16000 Hz
Source position ($\mathbf{r}_0$)	$(0, 0, 0)^T$ $(0.3, 0, 0)^T$ $(0, 0.3, 0)^T$ $(0, 0, 0.3)^T$
Sound velocity (c)	340 m/s
Total number of control points (M)	162
Radius of control points (r)	0.4 m
Total number of synthesis points (N)	162
Radius of synthesis points (R)	0.8 m
Unit normal vector ($\mathbf{n}_i$)	$\mathbf{r}_i$/|$\mathbf{r}_i$|
Neighbor distance (Δd_i)	0.002 m
Directivity of loudspeakers (D_i)	Omnidirectional, Bidirectional, Unidirectional, Shotgun

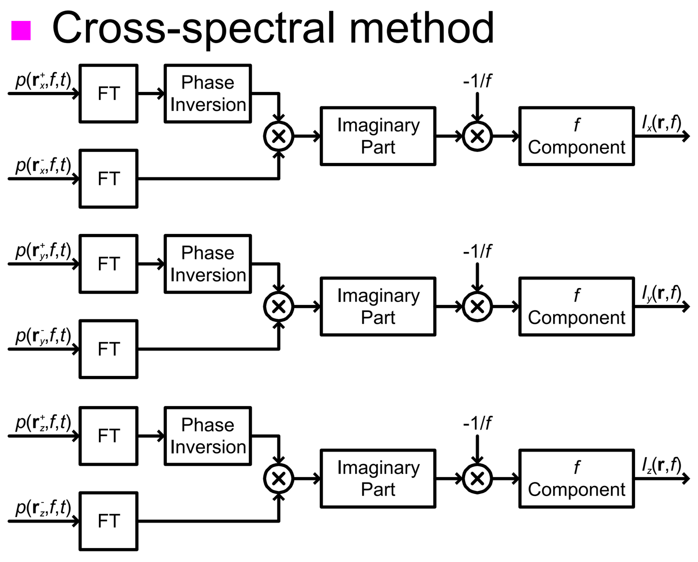
$$\mathbf{r}_i=(r\cos\theta_i\cos\phi_i\quad r\sin\theta_i\cos\phi_i\quad r\sin\phi_i)^T\ (i=1\ldots M)$$

$$\mathbf{r}=(R\cos\theta_j\cos\phi_j\quad R\sin\theta_j\cos\phi_j\quad R\sin\phi_j)^T\ (j=1\ldots N)$$

θ_i, ϕ_i: Azimuth and elevation angles of the ith control point
θ_j, ϕ_j: Azimuth and elevation angles of the jth synthesis point

Sound Intensity Vector

- Sound intensity vector $\mathbf{I}(\mathbf{r}, f)$
  - Correspond to the arrival direction of sound sources
- Cross-spectral method



$p(\mathbf{r}_x^+, f, t)$, $p(\mathbf{r}_x^-, f, t)$, $p(\mathbf{r}_y^+, f, t)$, $p(\mathbf{r}_y^-, f, t)$, $p(\mathbf{r}_z^+, f, t)$, $p(\mathbf{r}_z^-, f, t)$: Sound pressure at six points
$\mathbf{r}_x^+$, $\mathbf{r}_x^-$, $\mathbf{r}_y^+$, $\mathbf{r}_y^-$, $\mathbf{r}_z^+$, $\mathbf{r}_z^-$: Position vector at six points

$$\mathbf{r}_x^+=\mathbf{r}+(\Delta\ 0\ 0)^T,\quad \mathbf{r}_x^-=\mathbf{r}-(\Delta\ 0\ 0)^T,$$

$$\mathbf{r}_y^+=\mathbf{r}+(0\ \Delta\ 0)^T,\quad \mathbf{r}_y^-=\mathbf{r}-(0\ \Delta\ 0)^T,$$

$$\mathbf{r}_z^+=\mathbf{r}+(0\ 0\ \Delta)^T,\quad \mathbf{r}_z^-=\mathbf{r}-(0\ 0\ \Delta)^T$$

Δ=0.001 m
$\mathbf{I}(\mathbf{r}, f)=\{I_x(\mathbf{r}, f), I_y(\mathbf{r}, f), I_z(\mathbf{r}, f)\}^T$: Sound intensity vector

3.2. Simulation Results

- SNR of the RMSs of sound pressure
  - Performance of the sound pressure distribution

$$\mathrm{SNR}(f)=10\log_{10}\frac{\sum_{\mathbf{r}}\{p_0(\mathbf{r},f)\}^2}{\sum_{\mathbf{r}}\{p(\mathbf{r},f)-p_0(\mathbf{r},f)\}^2}$$

$p_0(\mathbf{r}, f)$: RMS of the sound pressure in the original sound field
$p(\mathbf{r}, f)$: RMS of the sound field in the reproduced sound field

$$p_0(\mathbf{r},f)=\sqrt{\frac{1}{T}\int_0^T\{p_0(\mathbf{r},f,t)\}^2dt}$$

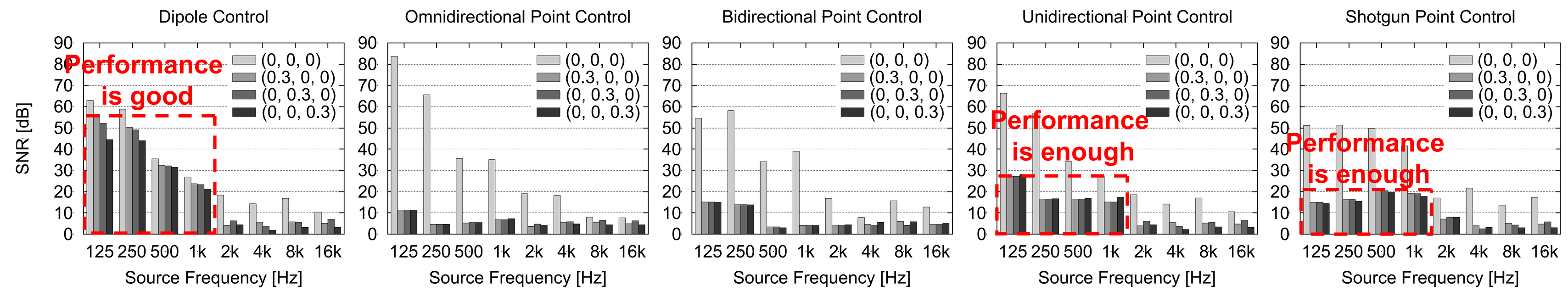
$$p(\mathbf{r},f)=\sqrt{\frac{1}{T}\int_0^T\{p(\mathbf{r},f,t)\}^2dt}$$

- Intensity direction error (IDE) $\theta(f)$
  - Performance of the arrival direction of sound sources

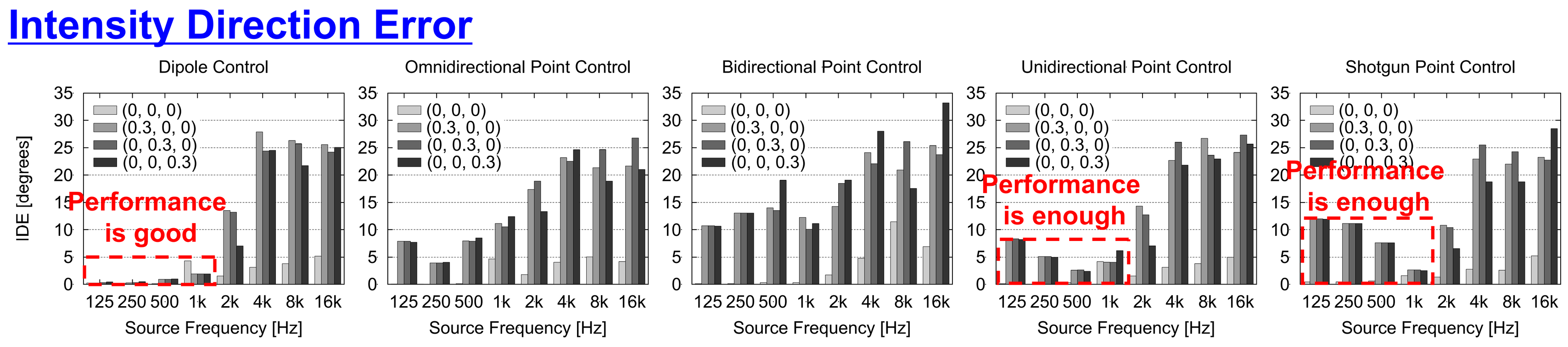
$$\theta(f)=\sqrt{\frac{1}{N}\sum_{\mathbf{r}}\left[\cos^{-1}\left\{\frac{\mathbf{I}(\mathbf{r},f)\cdot\mathbf{I}_0(\mathbf{r},f)}{|\mathbf{I}(\mathbf{r},f)\|\mathbf{I}_0(\mathbf{r},f)|}\right\}\right]^2}$$

$\mathbf{I}_0(\mathbf{r}, f)=\{I_{x0}(\mathbf{r}, f), I_{y0}(\mathbf{r}, f), I_{z0}(\mathbf{r}, f)\}^T$: Sound intensity vector in the original sound field
$\mathbf{I}(\mathbf{r}, f)=\{I_x(\mathbf{r}, f), I_y(\mathbf{r}, f), I_z(\mathbf{r}, f)\}^T$: Sound intensity vector in the reproduced sound field
N(=162): Total number of synthesis points

SNR of the RMSs of Sound Pressure



Intensity Direction Error



4. CONCLUSION

- **Near 3D sound field reproduction techniques based on wave field synthesis were proposed**
  - Two proposed methods...dipole control method and directional point control method
- Computer simulation was performed to evaluate the performance of two proposed methods
  - **The dipole control method performed very well**
  - **The directional point control method performed satisfactorily if the directivity of loudspeakers was unidirectional and shotgun**
- Future Works...Manufacture of microphone array and loudspeaker array